AP® Calculus AB
2003 Sample Student Responses

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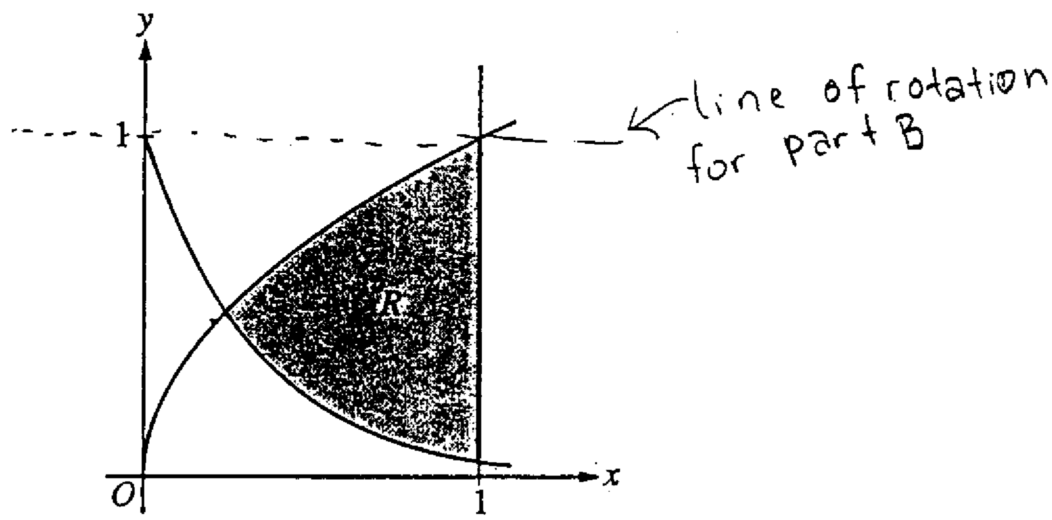
CALCULUS BC

SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.



Work for problem 1(a)

area of R = $A(R) = \int_{.239}^{1} \sqrt{x} - e^{-3x}\,dx$

$A(R) = .443$

intersect of $y=\sqrt{x}$ + $y=e^{-3x}$ is the lower bound for the integral!

$\sqrt{x} = e^{-3x}$

$x = .239$

Continue problem 1 on page 5.

Work for problem 1(b)

Volume of solid $= \pi \int_{.239}^{1} R^2 - r^2 \, dx$ $R = 1 - e^{-3x}$ $r = 1 - \sqrt{x}$

or

$V = \pi \int_{.239}^{1} (1 - e^{-3x})^2 - (1 - \sqrt{x})^2 \, dx$

$V = 1.424$

Work for problem 1(c)

$V = \int_{.239}^{1} h \cdot b \, dx$ $h = 5b$ $b = \sqrt{x} - e^{-3x}$

$V = \int_{.239}^{1} 5(\sqrt{x} - e^{-3x})(\sqrt{x} - e^{-3x}) \, dx$ $h = 5(\sqrt{x} - e^{-3x})$

$V = 1.554$

GO ON TO THE NEXT PAGE.

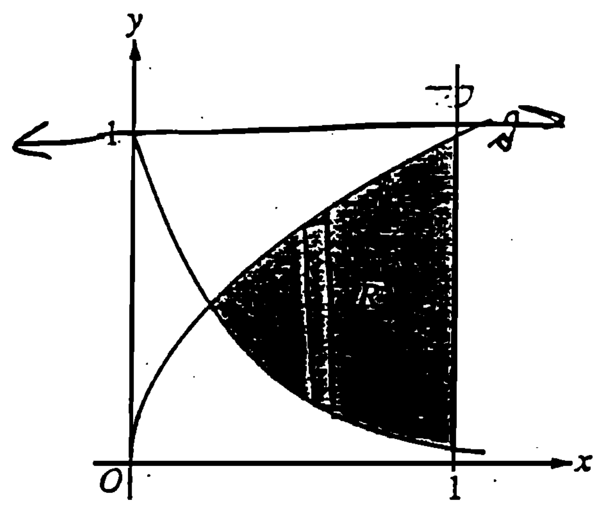
CALCULUS AB

SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.



Work for problem 1(a)

Area = $\int_{.24}^{1} \sqrt{x} - e^{-3x}\, dx = .443 \text{ units}^2$

Finding bounds: $x^{1/2} = e^{-3x}$

$x \approx .24$

Continue problem 1 on page 5.

Work for problem 1(b)

Found volume using washer method $V = \pi\int_a^b R^2 - r^2\, dr$

$V(x) = \pi \int_{.24}^{1} (1 - e^{-3x})^2 - (1 - x^{1/2})^2\, dx = 1.423$ units^3

Work for problem 1(c)

$\sqrt{x} - e^{-3x}$ = length of base

$5(\sqrt{x} - e^{-3x})$ = height

Area of Rec = base x height

$A = 5(\sqrt{x} - e^{-3x})^2$

To find Volume, integrate the area - use disc method

$V = 5\pi \int_{.24}^{1} (\sqrt{x} - e^{-3x})^2\, dx = 6.953$ units^3

GO ON TO THE NEXT PAGE.

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Work for problem 2(a)

(a) $v(t) = -(t+1)\sin\left(\frac{t^2}{2}\right)$ $v(2) = -3\sin(2) = -2.728$

$a(t) = -\sin\left(\frac{t^2}{2}\right) - (t+1)\cdot t\cos\left(\frac{t^2}{2}\right)$

$a(t) = -\sin\left(\frac{t^2}{2}\right) - t(t+1)\cos\left(\frac{t^2}{2}\right)$

$a(2) = -\sin(2) - 2(3)\cos(2)$

$\boxed{a(2) = 1.588}$

the speed of the particle is decreasing at $t=2$ because acceleration, the rate of change of the velocity is positive while velocity itself is negative, meaning that speed, the absolute value of velocity, is actually decreasing!

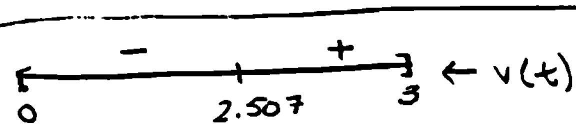
Work for problem 2(b)

(b) $v(t) = -(t+1)\sin\left(\frac{t^2}{2}\right)$

$t+1=0$ $\sin\left(\frac{t^2}{2}\right)=0$

~~$t=-1$~~ $t=2.507$

reject - not in interval



particle changes direction where $t = 2.507$

Continue problem 2 on page 7.

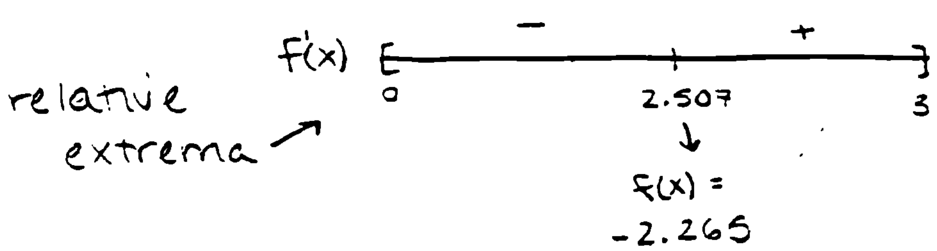
VV$_2$

Work for problem 2(c)

(c) Total distance traveled = $\int_0^3 |v(t)|\,dt$ = $\boxed{4.338}$

Work for problem 2(d)

Greatest distance between particle and origin will occur where the function $f(x) = 1 + \int_0^x v(t)\,dt$ achieves a relative max. or relative minimum.



distance between particle and origin = $1 + \int_0^{2.507} v(t)\,dt$ =

$\boxed{2.265}$

GO ON TO THE NEXT PAGE.

Work for problem 2(a)

$v(t) = -(t+1)\sin\left(\frac{t^2}{2}\right)$

$v'(t) = a(t) = -(t+1)\cos\left(\frac{t^2}{2}\right)\cdot t + -\sin\left(\frac{t^2}{2}\right)\cdot t$

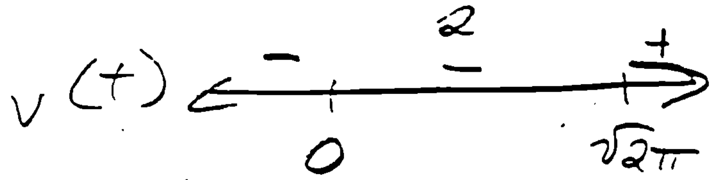
$a(2) = 1.588$

yes, the acceleration is positive. there For the slope of the velocity is positive.

Work for problem 2(b)

$v(t) = -(t+1)\sin\left(\frac{t^2}{2}\right) = 0$ at $0, + \sqrt{2\pi}$

$\frac{t^2}{2} = \pi$



The particle changes direction only at $\sqrt{2\pi}$ because the velocity changes From negative to positive.

Continue problem 2 on page 7.

 1L2

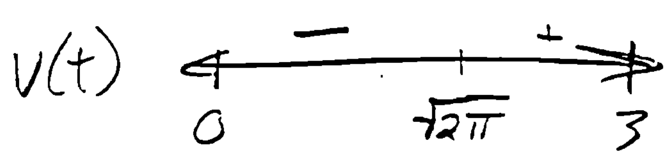
Work for problem 2(c)

$\int_0^{\sqrt{2\pi}} v(t)\,dt \quad \int_{\sqrt{2\pi}}^{3} v(t)\,dt$

$= 4.334$

Work for problem 2(d)

$\int_0^{\sqrt{2\pi}} v(t)\,dt = -3.265 + 1 = \boxed{-2.265}$



at $\sqrt{2\pi}$, the particle began to move towards the origin.

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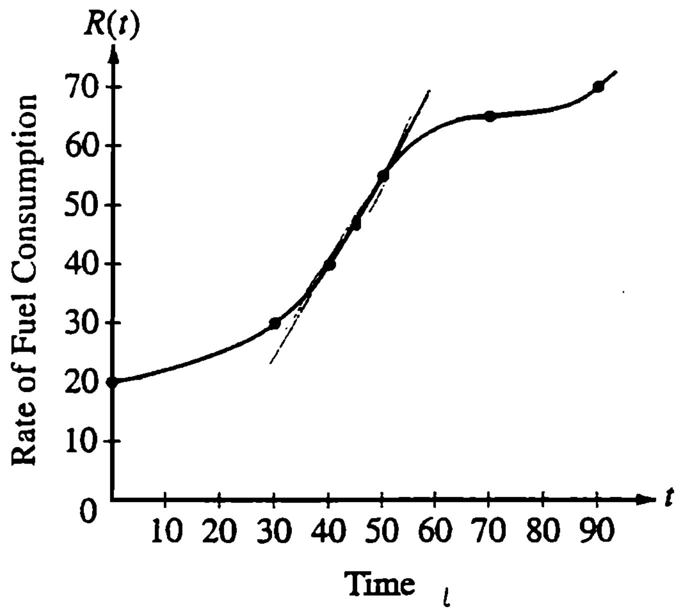


t (minutes)	$R(t)$ (gallons per minute)
0	20
30	30
40	40
50	55
70	65
90	70

Work for problem 3(a)

$R'(t) = \text{slope}$

$m = \frac{55-40 \text{ gal/min}}{50-40 \text{ min}} = 1.5$ gallons / minute^2

Work for problem 3(b)

$R''(45) = 0$

The value is zero because the rate of increase of fuel consumption is fastest at $t=45$. This would indicat a max point on $R'(45)$. As a result, $R''(45)$ is zero.

Continue problem 3 on page 9.

Work for problem 3(c)

$$\int_0^{90} R(t)\,dt = 30(20) + 10(30) + 10(40) + 20(55) + 2c$$
$$= 3700 \text{ gallons}$$

This approximation is less than the actual value because the rectangles are below the curve, and some of the fuel consumed is not included.

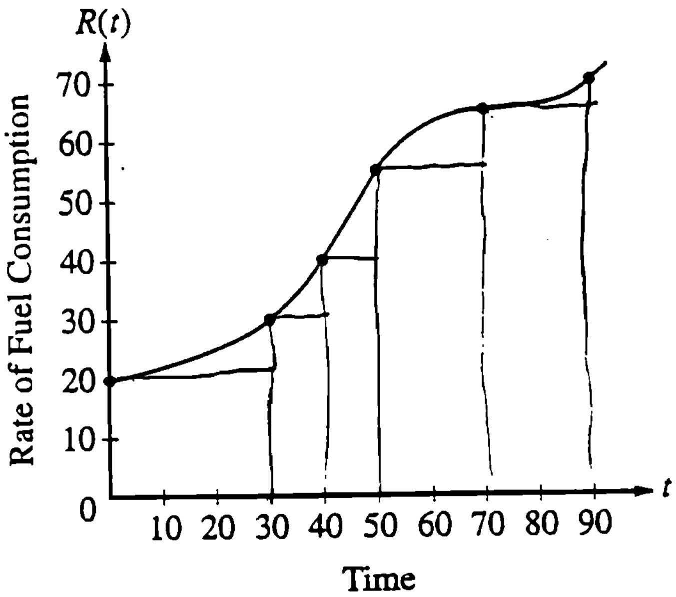
Work for problem 3(d)

$\int_0^b R(t)\,dt$ represents the number of gallons consumed from 0 to b minutes.

$\frac{1}{b}\int_0^b R(t)\,dt$ is the average value of fuelconsumption (in gallons) each minute from 0 to b minutes.

END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.



t (minutes)	$R(t)$ (gallons per minute)
0	20
30	30
40	40
50	55
70	65
90	70

Work for problem 3(a)

slope from $t=40$ to $t=50$:

$$m = \frac{55-40}{50-40}$$

$$= \frac{15}{10}$$

$$= \frac{3}{2}$$

$$\therefore R'(45) \approx \frac{3}{2}$$

Work for problem 3(b)

If at $t=45$, the rate is increasing fastest, the graph of t vs. $R(t)$ is changing concavity,

$$\therefore R''(45) = 0$$

Continue problem 3 on page 9.

I2

Work for problem 3(c)

$$\int_0^{90} R(t)\,dt \approx 30\times 20 + 10\times 30 + 10\times 40 + 20\times 55 + 20\cdot$$

$$\approx 3700$$

Since the function R(t) is increasing, this left Riemann sum is less than the actual value

---

Work for problem 3(d)

for $0<b\le 90$ min, $\int_0^b R(t)dt$ is the amount of fuel (in gallons) that the plane has consumed in the interval from t=0 and t=b minutes

$\frac{1}{b}\int_0^b R(t)\,dt$ is the average amount of fuel consumed per minute by the plane over the interval t=0 to t=b minutes. Since this is an average rate, the unit of measure is gallons per minute.

END OF PART A OF SECTION II

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NO CALCULATOR ALLOWED

C1

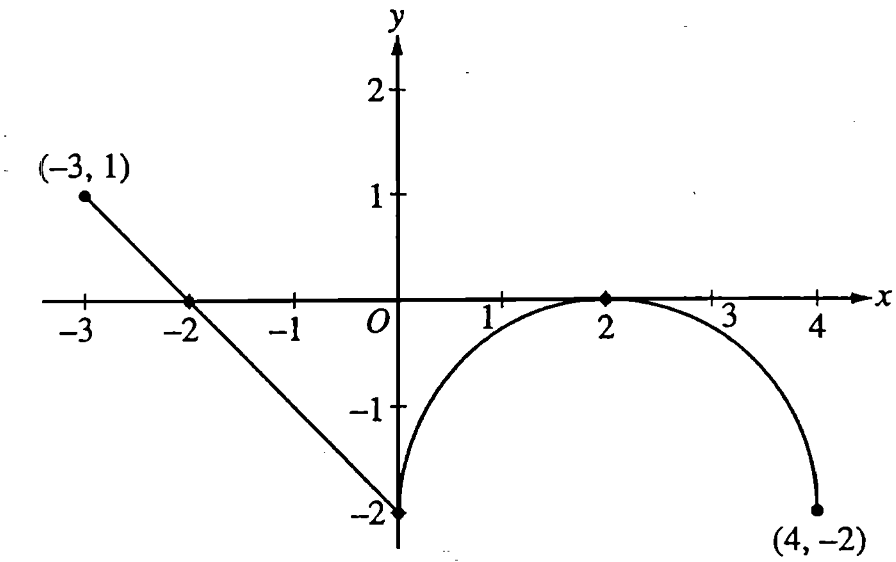
CALCULUS AB

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.



Graph of f'

Work for problem 4(a)

$-3 < x < -2$; $f'(x)$ is positive

Work for problem 4(b)

$x = 0, 2$;

x	$-3<x<0$	0	$0<x<2$	2	$2<x<4$
$f''(x)$	$-$	undefined	$+$	0	$-$

Continue problem 4 on page 11.

NO CALCULATOR ALLOWED

C2

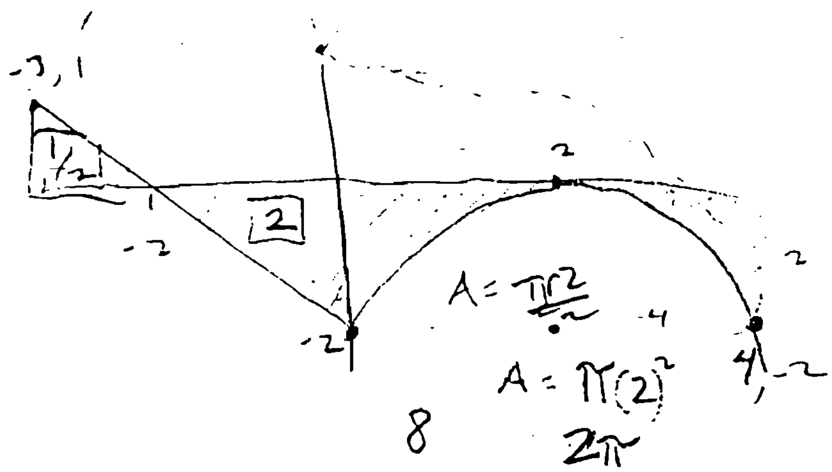
Work for problem 4(c)

point: (0, 3)

$f'(0) = -2$

$y - 3 = -2(x - 0)$

Work for problem 4(d)



$f(-3) = \int_0^{-3} f'(x)\,dx + 3$

$2 - \frac{1}{2} + 3 = 4.5$

$f(4) = \int_0^4 f'(x)\,dx + 3$

$-(8 - 2\pi) + 3$

$-8 + 2\pi + 3$

$= 2\pi - 5$

GO ON TO THE NEXT PAGE.

E1

NO CALCULATOR ALLOWED

CALCULUS BC

SECTION II, Part B

Time—45 minutes

Number of problems—3

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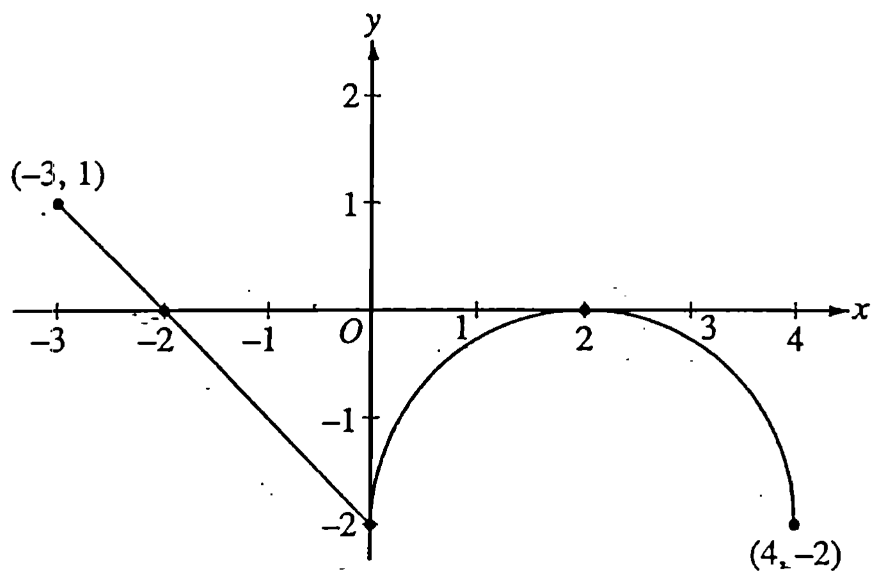

Graph of f'

Work for problem 4(a)

Increasing from x = −3 to x = −2
because the derivative is positive

Work for problem 4(b)

x = 0 and x = 2
these are the local maxima and minima of f'(x)

Continue problem 4 on page 11.

NO CALCULATOR ALLOWED

E2

Work for problem 4(c)

at (0, 3) slope is −2

$y - 3 = -2(x - 0)$

$y = -2x + 3$

Work for problem 4(d)

~~f(−5) = 1~~

$f(0) = 3$

$\int_0^{-3} f'(x) = -2 + .5$ or -1.5

$3 - 1.5$

so $f(-3) = 1.5$

$\int_0^4 f'(x) = -\frac{1}{2}\pi(2)^2 = -2\pi$

$f(0) - 2\pi = f(4)$

$f(4) = 3 - 2\pi$

GO ON TO THE NEXT PAGE.

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 B1

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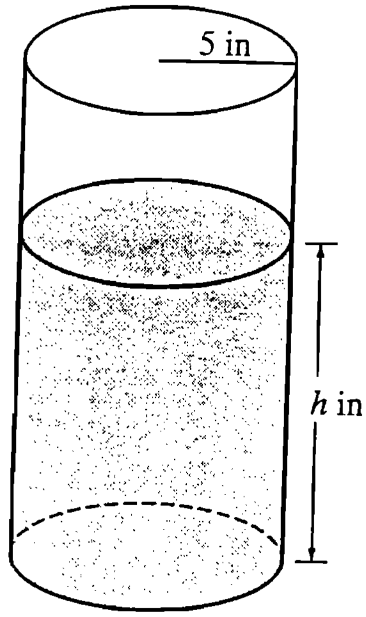


Work for problem 5(a)

$V = \pi r^2 h$

$\frac{dV}{dt} = -5\pi\sqrt{h} = \pi 25 \frac{dh}{dt}$

↓

$\frac{-5\sqrt{h}}{25} = 25\frac{dh}{dt} \div 25$

$-\frac{\sqrt{h}}{5} = \frac{dh}{dt}$

Continue problem 5 on page 13.

 B2

NO CALCULATOR ALLOWED

Work for problem 5(b)

~~$\int dh\, h^{-1/2} = \int -\frac{1}{5} dt$~~

$\int h^{-1/2}\, dh = \int -\frac{1}{5} dt$

$2h^{1/2} + C_1 = -\frac{1}{5}t + C_2$

$2\sqrt{h} = -\frac{1}{5}t + C_3$

$2\sqrt{17} = -\frac{1}{5}(0) + C_3$

$C_3 = 2\sqrt{17}$

$\frac{2\sqrt{h}}{2} = \frac{-\frac{1}{5}t + 2\sqrt{17}}{2}$

$\sqrt{h} = -\frac{t}{10} + \sqrt{17}$

$h = \left(-\frac{t}{10} + \sqrt{17}\right)^2$

Work for problem 5(c)

$h = 0$

~~$0 = \frac{t^2}{100} + \frac{2\sqrt{17}\,t}{10} + 17$~~

~~$\frac{2\sqrt{17}}{10} \pm \sqrt{\left(\frac{2\sqrt{17}}{10}\right)^2 - 4\ldots}$ over $(1/100)2$~~

$0 = \left(-\frac{t}{10} + \sqrt{17}\right)\left(-\frac{t}{10} + \sqrt{17}\right)$

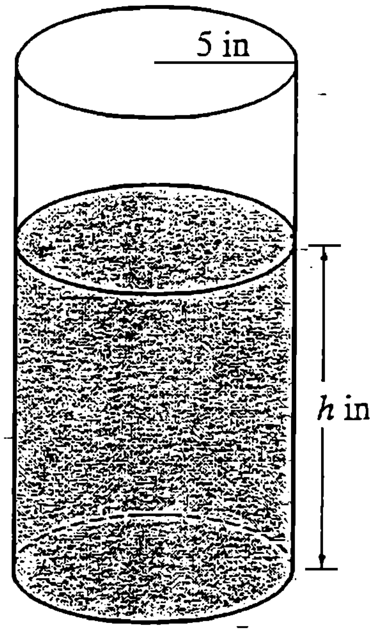
$\frac{t}{10} = \sqrt{17}$

$t = 10\sqrt{17}$ seconds

GO ON TO THE NEXT PAGE.

Y1

NO CALCULATOR ALLOWED



-h = depth in inches

$\frac{dV}{dt} = -5\pi\sqrt{h}$ in³/s

$V = \pi r^2 h$

$V = \pi(25)h$

Work for problem 5(a)

$\frac{dh}{dt} = -\frac{\sqrt{h}}{5}$

$\frac{dV}{dt} = -5\pi\sqrt{h}$

$V = \pi r^2 h \quad r = 5$

$V = 25\pi h$

$\frac{dV}{dt} = 25\pi \frac{dh}{dt}$

$\frac{-5\pi\sqrt{h}}{25\pi} = \frac{25\pi}{25\pi} \cdot \frac{dh}{dt}$

$\boxed{-\frac{\sqrt{h}}{5} = \frac{dh}{dt}}$

Continue problem 5 on page 13.

$\frac{1}{2}$

NO CALCULATOR ALLOWED

Work for problem 5(b)

$$\frac{dh}{dt} = \frac{-\sqrt{h}}{5} \cdot dt$$

$$\frac{1}{\sqrt{h}} \cdot dh = \frac{-\sqrt{h}}{5} \cdot dt \cdot \frac{1}{\sqrt{h}}$$

$$\int h^{-1/2} dh = \int -\frac{1}{5}\, dt$$

$$2h^{1/2} = -\frac{1}{5}t + C$$

$$2\sqrt{h} = \frac{-1}{5}t + C \qquad h = 17 \text{ at time } t = 0$$

$$2\sqrt{17} = -\frac{1}{5}(0) + C$$

$$C = 2\sqrt{17}$$

$$(\sqrt{h})^2 = \left(\frac{1}{10}t + \sqrt{17}\right)^2$$

$$\frac{2\sqrt{h}}{2} = \frac{\frac{1}{5}t + 2\sqrt{17}}{2}$$

$$h = \left(\frac{t}{10} + \sqrt{17}\right)^2$$

Work for problem 5(c)

$V = 0 = 25\pi h$

$h = 0$

$$0 = \left(\frac{t}{10} + \sqrt{17}\right)^2$$

$$0 = \frac{t}{10} + \sqrt{17}$$

$$\cdot 10 \; -\sqrt{17} = \frac{t}{10} \cdot 10$$

$t = 10\sqrt{17}$ seconds

GO ON TO THE NEXT PAGE.

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Work for problem 6(a)

yes, f(3) is 2, which is the same as the limits as x approaches 3 from either side.

Work for problem 6(b)

$$\frac{\int_0^3 \sqrt{x+1}\,dx + \int_3^5 (5-x)\,dx}{5}$$

$$\frac{\left[\frac{2}{3}(x+1)^{3/2}\right]_0^3 + \left[5x-\frac{1}{2}x^2\right]_3^5}{5}$$

$$\frac{\frac{2}{3}(4)^{3/2}-\frac{2}{3}+25-\frac{1}{2}25-(15+\frac{1}{2}9)}{5}$$

$$\frac{\frac{2}{3}8-\frac{2}{3}+10-\frac{16}{2}}{5}$$

$$\frac{16}{15}-\frac{2}{15}+2-\frac{8}{5}$$

$$\frac{14}{15}+2-\frac{8}{5}$$

$$\frac{14}{15}+\frac{30}{15}-\frac{24}{15}$$

$$\frac{20}{15}$$

$$\frac{4}{3}$$

Continue problem 6 on page 15.

NO CALCULATOR ALLOWED

Work for problem 6(c)

$g' = k\frac{1}{2\sqrt{x+1}}$

m

$k\frac{1}{2\sqrt{3+1}} = m$ ← slopes must be equal at 3

functions must be equal at 3

$\frac{1}{4}k = m$

$k(2) = m(3) + 2$

$m = \frac{2k-2}{3}$

$\frac{1}{4}k = \frac{2k-2}{3}$

$k = \frac{8k-8}{3}$

$k = \frac{8}{3}k - \frac{8}{3}$

$-\frac{5}{3}k = -\frac{8}{3}$

$k = \frac{24}{15} = \frac{8}{5}$

$\frac{1}{4}\left(\frac{8}{5}\right) = \frac{8}{20} = \frac{2}{5}$

$k = \frac{8}{5}$ $m = \frac{2}{5}$

END OF EXAMINATION

THE FOLLOWING INSTRUCTIONS APPLY TO THE BACK COVER OF THIS SECTION II BOOKLET.

- **MAKE SURE YOU HAVE COMPLETED THE IDENTIFICATION INFORMATION AS REQUESTED ON THE BACK OF THIS SECTION II BOOKLET.**
- **CHECK TO SEE THAT YOUR AP NUMBER APPEARS IN THE BOX(ES) ON THE BACK COVER.**
- **MAKE SURE THAT YOU HAVE USED THE SAME SET OF AP NUMBER LABELS ON ALL AP EXAMINATIONS YOU HAVE TAKEN THIS YEAR.**

Work for problem 6(a)

$f(x) = \begin{cases} \sqrt{x+1} & 0 \le x \le 3 \\ 5-x & 3 < x \le 5 \end{cases}$

$f(3) = \begin{cases} \sqrt{3+1} = \sqrt{4} = 2 \\ 5-3 = 2 = 2 \end{cases}$

f is continuous at $x=3$ because the $\lim_{x \to 3} f(x) = 2$ and $f(3) = 2$.

Work for problem 6(b)

$u = x+1$
$du = 1$
$u(3) = 4$
$u(0) = 1$

Average value $= \frac{1}{5-0}\int_0^5 f(x)\,dx$

The average value is $4/3$ on $0 \le x \le 5$.

$= \frac{1}{5}\left(\int_0^3 \sqrt{x+1}\,dx + \int_3^5 (5-x)\,dx\right)$

$= \frac{1}{5}\left(\int_1^4 \sqrt{u}\,du + \int_3^5 (5-x)\,dx\right)$

$= \frac{1}{5}\left(\frac{2}{3}u^{3/2}\Big|_1^4 + 5x - \frac{1}{2}x^2\Big|_3^5\right)$

$= \frac{1}{5}\left(\left(\frac{2}{3}(4)^{3/2} - \frac{2}{3}(1)^{3/2}\right) + \left(\left(5(5) - \frac{1}{2}(5)^2 - 5(3) - \frac{1}{2}(3^2)\right)\right)\right)$

$= \frac{1}{5}\left(\left(\frac{2}{3}(8) - \frac{2}{3}\right) + \left(\left(25 - \frac{25}{2}\right) - \left(15 - \frac{9}{2}\right)\right)\right)$

$= \frac{1}{5}\left(\left(\frac{16}{3} - \frac{2}{3}\right) + \left(\left(\frac{50}{2} - \frac{25}{2}\right) - \left(\frac{30}{2} - \frac{9}{2}\right)\right)\right)$

$= \frac{1}{5}\left(\left(\frac{14}{3}\right) + \left(\frac{25}{2} - \frac{21}{2}\right)\right)$

Continue problem 6 on page 15.

$= \frac{1}{5}\left(\frac{14}{3} + \frac{4}{2}\right) = \frac{1}{5}\left(\frac{14}{3} + 2\right) = \frac{1}{5}\left(\frac{14}{3} + \frac{6}{3}\right) = \frac{1}{5}\left(\frac{20}{3}\right) = \frac{20}{15} = \frac{4}{3}$

NO CALCULATOR ALLOWED

V_2

Work for problem 6(c)

$g(x) = \begin{cases} k\sqrt{x+1} & 0 \le x \le 3 \\ mx+2 & 3 < x \le 5 \end{cases}$

$g(3) = \begin{cases} k\sqrt{3+1} = k\sqrt{4} = 2k \\ 3m+2 \end{cases}$

$\frac{2k}{2} = \frac{3m+2}{2}$

$k = \frac{3m+2}{2}$

$\frac{3m+2}{2}\sqrt{x+1} = 0$

$\frac{3m+2}{2}\sqrt{3+1} = 0$

$\frac{3m+2}{2}\sqrt{4} = 0$ $\quad \frac{3m+2}{2}(2) = 0$ $\quad 3m+2 = 0$

$3m+2 = 0$
$\quad -2 \quad -2$

$\frac{3m}{3} = \frac{-2}{3}$

$m = -2/3$

$k = \frac{3(-2/3)+2}{2}$

$k = \frac{-2+2}{2} = 0$

$k = 0$

END OF EXAMINATION

THE FOLLOWING INSTRUCTIONS APPLY TO THE BACK COVER OF THIS SECTION II BOOKLET.

- **MAKE SURE YOU HAVE COMPLETED THE IDENTIFICATION INFORMATION AS REQUESTED ON THE BACK OF THIS SECTION II BOOKLET.**
- **CHECK TO SEE THAT YOUR AP NUMBER APPEARS IN THE BOX(ES) ON THE BACK COVER.**
- **MAKE SURE THAT YOU HAVE USED THE SAME SET OF AP NUMBER LABELS ON ALL AP EXAMINATIONS YOU HAVE TAKEN THIS YEAR.**